Roll No. ....

D–984

M. A./M. Sc. (Fourth Semester) (Main/ATKT)

EXAMINATION, May-June, 2020

MATHEMATICS

Paper Second

(Partial Differential Equations and Mechanics—II)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. The 'Eikonal' equation from geometric optics is the PDE :

(a) $x.\mathrm{D}u + f(\mathrm{D}u) = u$

(b) $|\mathrm{D}u| = 1$

(c) $u_t + x(\mathrm{D}u) = 0$

(d) $\mathrm{D}_p\mathrm{F} = b(x)$

2. $-\frac{d}{ds}\left(\mathrm{D}_q\mathrm{L}\left(\dot{x}(s), x(s)\right)\right) + \mathrm{D}_x\mathrm{L}\left(\dot{x}(s), x(s)\right) = 0$

$$(0 \le s \le t)$$

is known as :

(a) Hamilton's ODE

(b) Conservation law

(c) Euler-Lagrange equations

(d) Wave equation

3. $x.\mathrm{D}u + f(\mathrm{D}u) = u$ is known as :

(a) Heat equation

(b) Clairaut's equation

(c) Porous medium equation

(d) None of the above

4. Right hand side of the following equation is known as :

$$u(x,t) = \min_{y \in \mathrm{R}^n}\left\{t\,\mathrm{L}\left(\frac{x-y}{t}\right) + g(y)\right\} \quad \left(x \in \mathrm{R}^n, t > 0\right)$$

(a) Hopf-Lax formula

(b) Legendre transform

(c) Semiconcavity

(d) Telegraph equation

5. Equation $u_t - \Delta\left(u^y\right) = 0$ in $\mathrm{R}^n \times (0, \infty)$ is known as :
   (a) Hamilton ODE
   (b) Porous medium equation
   (c) Potential function solution
   (d) Laplace equation

6. Equation $u(x, t) = v(x - \sigma t)$ $(x \in \mathrm{R},\ t \in \mathrm{R})$ is known as :
   (a) Exponential equation
   (b) Fourier transform
   (c) Travelling wave
   (d) Bessel potentials

7. Equation :

$$\hat{u}(y) = \frac{\mathrm{L}}{(2\pi)^{n/2}} \int_{\mathrm{R}^n} e^{i\,x.y} u(x)\, dx$$

$$\left(y \in \mathrm{R}^n, u \in \mathrm{L}^1\left(\mathrm{R}^n\right)\right)$$

   is known as :
   (a) Fourier transform
   (b) Inverse Fourier transform
   (c) Plane wave equation
   (d) None of the above

8. The equation

$$u_t - u_{xx} = f(u) \text{ in } \mathrm{R} \times (0, \infty)$$

   is known as :
   (a) Airy's equation
   (b) Burger equation
   (c) K*d*V equation
   (d) Scalar reaction-diffusion equation

9. Taylor expansion about x_0 is :
   (a) $f(x) = \sum_{\alpha} f(x - x_0)^{\alpha} \left(|x - x_0| < r\right)$
   (b) $f(x) = \sum_{\alpha} \frac{1}{\lfloor\underline{x}} f(x - x_0)^{\alpha} \left(|x - x_0| < r\right)$
   (c) $f(x) = \sum_{\alpha} \frac{1}{\lfloor\underline{x}} \mathrm{D}^{\alpha} f(x_0)(x - x_0)^{\alpha} \left(|x - x_0| < r\right)$
   (d) $f(x) = \sum_{\alpha} \mathrm{D}^{\alpha} f(x)(x - x_0)^{\alpha} \left(|x - x_0| < r\right)$

10. Expansion is known as :

$$f = \sum_{\alpha} f_{\alpha}\, x^{\alpha}$$

   (a) Power series
   (b) Multi-indices
   (c) Majorizes
   (d) None of the above

11. kth-order quasilinear PDE is :

(a) $\Sigma\left(D^k v, ...u, x\right) + \left(D^{k-1}u, ...u, x\right) = 0$

(b) $\sum_{|\alpha|=k} a_\alpha \left(D^{k-1}u, ...u, x\right) D^\alpha u +$

$$a_0 \left(D^{k-1}\, u......u, x\right) = 0$$

(c) $\Sigma\, a_\alpha \left(D^k u.......u, x\right) + a_0 \left(D^{k-1}\, u......u, x\right) = 0$

(d) $\sum_{|\alpha|=k} \left(D^{k-1}u, ...u, x\right) + a_0 \left(D^{k-1}\, u......u, x\right) = 0$

12. The second-order hyperbolic PDE is :

(a) $u_t - \Sigma\, a^{kl}\left(x\right) u_{x_k} x_l = 0 \text{ in } R^n \times \left(0, \infty\right)$

(b) $u_{tt} - \Sigma\, a^{kl}\left(x\right) = 0 \text{ in } R^n \times \left(0, \infty\right)$

(c) $u_{tt} - \sum_{k,l=1}^{n} a^{kl}\left(x\right) u_{x_k} x_l = 0 \text{ in } R^n \times \left(0, \infty\right)$

(d) $u_t - \sum_{k,l=1}^{n} a^{kl}\left(x\right) = 0 \text{ in } R^n \times \left(0, \infty\right)$

13. Line integral $W = \int_{t_1}^{t_2} L\, dt$, where $L dt$ is called :

(a) Action

(b) Interval

(c) Elementary action

(d) None of the above

14. The following differential equations are known as

$$\frac{dq_j}{dq_1} = \frac{\partial k}{\partial p_j}$$

$$\frac{\partial p_j}{dq_1} = -\frac{\partial k}{\partial q_j} \quad (j = 2, 3, ..., n)$$

(a) Jacobi equation

(b) Hamilton's principle

(c) Lagrange bracket

(d) Whittaker's equation

15. Fermat's principle in geometrical optics is :

(a) $\Delta\left(t_2 - t_1\right) = 0$

(b) $\int_{t_1}^{t_2} 2T\, dt = 0$

(c) $\Delta\left(t_2 - t_1\right) \neq 0$

(d) $H = T + V$

16. Lagrange's equation of motion for conservative, holonomic dynamical system is :

(a) $\frac{\partial L}{\partial q_k} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = Q_j$

(b) $\frac{\partial T}{\partial q_k} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) = Q_j$

(c) $\frac{\partial L}{\partial q_k} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = 0$

(d) $\frac{\partial T}{\partial q_k} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) = 0$

17. The second form of Jacobi's theorem states that :

(a) $H\left(\frac{\partial w}{\partial q_i}, q_i\right) = \alpha_1$

(b) $H\left(\frac{\partial w}{\partial q_i}\right) = -\alpha_1$

(c) $H\left(\frac{\partial w}{\partial q_i}, q_i\right) = 0$

(d) $H\left(\frac{\partial w}{\partial q_i}, q_i\right) \neq \alpha_1$

18. The first form of Hamilton theorem is :

(a) $S(t) = \int L\,dt$ + constant

(b) $S(t) = \int^{t} L\,dt$

(c) $K = H + \frac{\partial f}{\partial t}$

(d) None of the above

19. The solution to Hamilton-Jacobi equation will be in the form :

$$S = -\alpha_1 t + W(q_1, q_2, \ldots\ldots, q_n, \alpha_1, \alpha_2, \ldots\ldots, \alpha_n)$$

where W is known as :

(a) Hamilton-Jacobi equation

(b) Hamilton principle function

(c) Hamilton characteristic function

(d) Canonical function

20. The correct fundamental Lagrange bracket is :

(a) $\{q_i, p_j\} = \delta_{ij}, \delta_{ij} \begin{matrix} = 1, & \text{if } i = j \\ = 0, & \text{otherwise} \end{matrix}$

(b) $\{q_i, q_j\} = \delta_{ij}$

(c) $\{p_i, p_j\} = \delta_{ij}$

(d) $\{q_i, p_j\} \neq \delta_{ij}$

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Explain the F-linear for non-linear first order partial differential equation :

$$F(Du, u, x) = 0$$

2. Write statement of Lax-Oleinik formula.
3. Write properties of Fourier transform.
4. Define heat and wave equation under similarity solution.
5. Define Real analytic functions.
6. Write statement of Hamilton's principle.
7. Write statement for first form of Jacobi's theorem.
8. Define canonical transformation.

Section—C 3 each

(Short Answer Type Questions)

***Note* :** Attempt all questions.

1. Define Laplace transform with example.
2. State and prove Majorants.
3. Derive Legendre transform.
4. Write a short note on non-characteristic surfaces.
5. Derive Hamilton's principle from Lagrange's equation.

6. Prove the properties of contact transformation :

(i) $\dfrac{\partial q_i}{\partial Q_j} = \dfrac{\partial P_j}{\partial p_i}$

(ii) $\dfrac{\partial q_i}{\partial P_j} = \dfrac{-\partial Q_j}{\partial p_i}$

7. Derive Hamilton-Jacobi equation for Hamilton's characteristic function.
8. A particle is thrown vertically upward with an initial velocity u against the attraction due to gravity. Write down the Hamilton-Jacobi equation for the motion and general solution of the equation of motion.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt any *four* questions.

1. Derive a functional identity.
2. Derive asymptotes in L^{∞} – norm.
3. State and prove the Plancherel's theorem.
4. Derive oscillating solutions for wave equation.
5. Derive Barenblatt's solution to the porous medium equation.
6. The transformation equations between two sets of co-ordinates are :

$$Q = \log\left(1 + \sqrt{q} \cos p\right)$$

$$P = 2\left(1 + \sqrt{q} \cos p\right)\sqrt{q} \sin p$$

Then show that these transformations are canonical if q, p are canonical.

7. Discuss the motion of a particle in one dimension H-J method.

8. If $u_l, l = 1, 2, ..., 2n$ forms a set of $2n$ independent functions, such that u is a function of $2n$ co-ordinates $q_1, q_2 ......, q_n$, $p_1, p_2, ..., p_n$, then prove that :

$$\sum_{l=1}^{2n} \{u_l, u_i\}\{u_l, u_j\} = \delta_{ij}$$